SEMESTER-I

PERIOD-I

M10CH1

TOPIC 1

Sets and Operations on Sets

1.1 SET CONCEPT

ACTIVITY 1

As a whole class, arrange yourself in two groups:

(*a*) Group of girls (*b*) Group of boys

Is it possible to make a group of students who weight more than 30 kg? Why?

Discuss the definition of set.

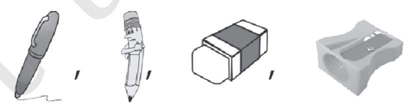
Definition of Set

A well-defined collection of objects is called a set. Each object is an element or a member of the set. All the elements of a set are written within the curly brackets { }.

Notation of Set

We use capital letters such as A, B, C, D, ... to name the sets.

For example:

1. A = {  } is a set of geometrical instruments.
2. B = {  } is a set of vegetables.
3. M = {January, February, ..., December} is a set of months of a year.

Let us write a set of days in a week. We write it as:

{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

This is well-defined and therefore, is a set.

Now, let us write a set of good football players. We write different sets with different football players, because there is a no particular way to decide who good football players are. Therefore, it is not well-defined and is not a set.

Note: The collection of objects or things should be clearly defined to form a set.

Example 1. *Which of the following are sets?*

(*a*) *{10, 20, 30, 40, 50}*

(*b*) *{prime numbers}*

(*c*) *{all even natural numbers each less than 20 and more than 16}*

(*d*) *The collection of all good wrestlers of the world*

(*e*) *{factors of 24}*

(*f*) *The collection of ten most talented writers of Liberia*

Solution. (*a*), (*b*), (*c*), (*e*)

Note: (*d*) and (*f*) are not well-defined and therefore they are not sets.

Cardinal Number

ACTIVITY 2

Consider the activity 1 again.

Count and write the number of girls.

Count and write the number of boys.

The number of elements in set A is called its cardinal number. We denote it as $n(A)$.

For example:

Look at the set of fruits below:

A = 

Count the number of fruits.

That is, 5.

So, $n(A) = 5$

Example 2. *Determine the cardinality of the set A = {1, 3, 5, 7}*

Solution. It has 4 elements, so, $\boldsymbol{n(A) = 4}$.

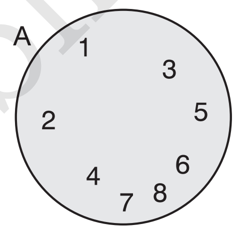
1.2 VENN DIAGRAMS

We can also represent sets using Venn diagrams. In a Venn diagram, the sets are represented by shapes; usually circles or ovals or rectangles. The elements are represented by points in the interior of the shapes.

Example 3. *Given the set A = {1, 2, 3, 4, 5, 6, 7, 8}, draw and label a Venn diagram to represent the set A.*

Solution. Set A = {1, 2, 3, 4, 5, 6, 7, 8}

Draw a circle or an oval. Label it A. Put the elements in A.



Elements of a Set

ACTIVITY 3

In a group of 5 pupils, consider the following sets.

Set of girls (G) = {Islah, Emine, Aaliyah, Shelia, Ella}

Set of boys (B) = {Daniel, Felix, Albert, Amos, William}

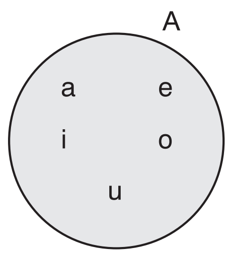
Discuss the following in your group:

1. Does Islah belong to set G?
2. Does William belong to set B?
3. Does Sonia belong to set G?
4. Does Samuel belong to set B?

When an element is a member of a given set, we say that it belongs to that set. We use the symbol $\in$ to show it.

For example:

Suppose set A = {a, e, i, o, u}



Is a an element of set A?

Is c an element of set A?

Here, we observe that a is an element of set A, but c is not an element of set A.

Therefore, $a \in A$

and $c \notin A$.

When an element is not a member of a set, we say that it does not belong to that set. We use the symbol $\notin$ to show it.

Example 4. (*a*) *Write* $\in$ *or* $\notin$ *in the spaces provided.*

(*i*) *3 ___ {multiples of 3}* (*ii*) *x ___ {a, e, i, o, u}*

(*b*) *Use the given Venn diagram and write* $\in$ *or* $\notin$ *in the spaces provided.*

(*i*) *m ___ X* (*ii*) *q ___ X.*

Solution. (*a*) (*i*) $3 \in$ {multiples of 3} (*ii*) $x \notin \{a, e, i, o, u\}$

(*b*) (*i*) $m \in X$ (*ii*) $q \notin X$

EXERCISE 1.1

1. Which of the following are sets?

(*a*) {10, 9, 8, 7, 6, 5, 4, 3, 2, 1}

(*b*) {2, 4, 6, 8, 10, ..., 100}

(*c*) A collection of all students in the world

(*d*) A collection of the most dangerous animals of the world

(*e*) {factors of 28}

2. Write down all the elements of the following sets.

(*a*) {7, 9, 11, 25, 26}

(*b*) $\{a, e, i, o, u\}$

(*c*) {the odd numbers between 8 and 25}

(*d*) {the even natural numbers less than 20}

3. Determine the cardinality of each of the following sets.

(*a*) A = {1, 2, 3, 4, 5, 6, 7}

(*b*) B = {2, 4, 6, 8, 10, 12, 14, 16, 20, 22}

(*c*) C = {all factors of 45}

(*d*) P = {all odd natural numbers}

4. If a set has 7 elements, find the cardinality of the set.
5. In the above question 2, represent each set using Venn diagram.
6. Copy and fill in the blanks using the symbol $\in$ or $\notin$.
   (*a*) *a* ___ {*a*, *b*, *c*, *d*, *e*}
   (*b*) 5 ___ {even prime numbers}
   (*c*) 22 ___ {1, 2, 3, …., 70}
   (*d*) 18 ___ {factors of 54}

1.3 TYPES OF SETS

ACTIVITY 4

In the same group as you did activity 3, discuss the following:

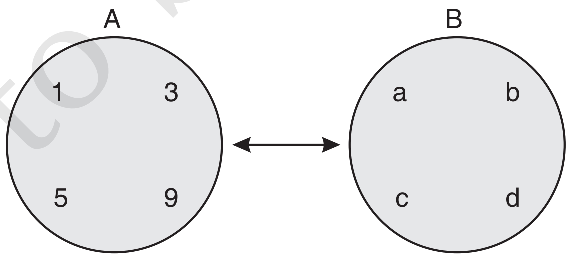
1. Is the number of elements in set G equal to the number of elements in set B? What are these types of sets called?
2. Are the elements in set G exactly the same with the elements in set B? What are these types of sets called?

Equivalent Sets

Sets having equal number of members or elements are called *equivalent sets.*

Consider the sets A = {1, 3, 5, 9} and B = {*a*, *b*, *c*, *d*}. Set A has 4 members. Set B has 4 members. Sets A and B both have the same number of members but do not have exactly the same members. So, they are equivalent sets. We write, Set A $\leftrightarrow$ Set B.

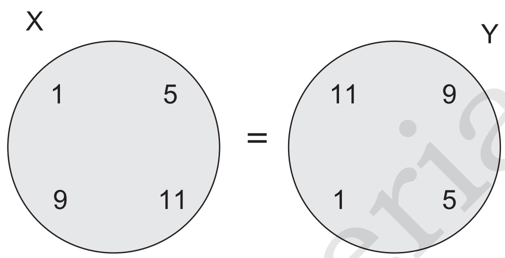
Let us represent them using Venn diagrams.



Equal Sets

Sets having exactly the same members or elements are called *equal sets.*

Consider the sets X = {1, 5, 9, 11} and Y = {11, 9, 1, 5}. Set X has 4 members. Set Y has 4 members. Sets X and Y both have exactly the same members. So, they are equal sets. We write, Set X = Set Y.



Let us represent them using Venn diagrams (see the diagram).

Note: Sets having unequal number of members or elements are called *unequal sets.*

Example 5. *State equivalent, equal or unequal sets for each of the following:*

(*a*) *A* = {*1, 2, 5, 6*} *and B* = {*2, 5, 6, 1*}
(*b*) *A* = {*7, 8, 9, 0*} *and B* = {*1, 2, 3, 4*}
(*c*) *X* = {*a, b, c*} *and Y* = {*boat, ship*}

Solution.

(*a*) Set A and set B are equal and equivalent sets.
(*b*) Set A and set B are equivalent and unequal sets.
(*c*) The given sets are unequal sets.

Empty or Null Set

ACTIVITY 5

Is it possible to make a set of persons who have:

(*a*) Three legs (*b*) 3 eyes (*c*) 25 fingers

What do you call these types of sets? Discuss in pairs.

A set that has no member or element is called an *empty* or *null set.*

For example:

X = {The months with 32 days}

Since no month of a year has 32 days,
set X = {The months with 32 days} is an empty set.
The symbol '{ }' or 'ϕ' is used to denote an empty set.
A set which has at least one element is called a *non-empty set.*

Note: Number 0 does not represent the empty set.
{ϕ} and {0} are also not empty sets.

Example 6. *Find the empty sets in the following:*

(*a*) *The set of dogs with 6 legs.*

(*b*) *The set of squares with 4 sides.*

(*c*) *The set of cars with 300 doors.*

Solution. Empty sets are: (*a*) and (*c*)

Finite Set

A set with *limited* number of members or one whose last member is known is called a *finite set.*

For example:

(*a*) A = {1, 2, 3, 4, 5, 6, 7, 8}

(*b*) B = {*a*, *b*, *c*, *d*, *e*, *f*, *k*}

(*c*) C = {Men living presently in different parts of the world}

(*d*) D = {Pages in a book}.

We observe that A contains 8 members and B contains 7 members.

How many members does C contain?

It is some natural number which may be quite a big number. D will also contain some fixed number as it's members.

All these sets are *finite.*

Note that the members of a finite set can be counted.

Infinite Set

A set with *unlimited* number of elements or one whose last element is not known is called an *infinite set.*

For example:

(*a*) N = {1, 2, 3, 4, 5, ...} is the set of natural numbers

(*b*) W = {0, 1, 2, 3, 4, ...} is the set of whole numbers

(*c*) E = {2, 4, 6, 8, 10, ...} is the set of even natural numbers.

(*d*) G = {the set of points on a number line}

All these sets are *infinite.*

Note that the members of an infinite set cannot be counted as it continues infinitely.

Example 7. *State which of the following sets are finite or infinite:*

(*a*) {*days of the week*} (*b*) {*odd natural numbers*}

(*c*) {*prime numbers*} (*d*) {*numbers which are multiple of 7*}

(*e*) {*animals living on the earth*}

(*f*) {*months of a year not having 31 days*}.

Solution. (*a*) Finite (*b*) Infinite

(*c*) Infinite (*d*) Infinite

(*e*) Finite (*f*) Finite.

Note: **1.** A set which is empty or consists of a definite number of members is called *finite* otherwise, the set is called *infinite*.

2. A unit set is also called *singleton set*.

3. A unit set is a finite set.

Universal Set

Consider the following set.

U = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

This is a set of days of a week and is called a universal set.

The set of all members at a time make a universal set.

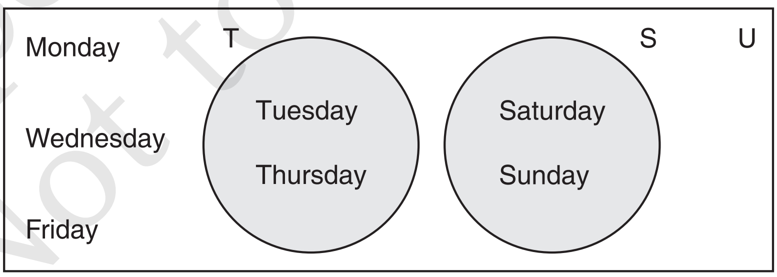
We use the symbol ξ or U to represent it.

We can form several sets using this universal set.

(*a*) A set of days starting with alphabet T = {Tuesday, Thursday}

(*b*) A set of days starting with alphabet S = {Saturday, Sunday}

Let us represent them in a Venn diagram.



Observe that from one universal set, we can form many subsets (*Defined further*).

Example 8. *Three sets are given:*

A = {4, 5, 6, 10, p, q, r}, B = {0, 1, 2, 4, 5, a, b},

C = {0, 1, 2, 4, 5, 10, a, b, p, q, r}

Is set C the universal set of sets A and B? If not, why?

Solution. No, because 6 is an element of A but not of C.

But C is a universal set of set B because all elements of set B lie in set C.

Example 9. *Consider a universal set of numbers from 1 to 10.*

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Write several sets using the above set.

(*a*) *A set of even numbers up to 10*

(*b*) *A set of odd numbers up to 10*

(*c*) *A set of numbers divisible by 3 up to 10.*

Solution.

(*a*) A set of even numbers up to 10 = {2, 4, 6, 8, 10}

(*b*) A set of odd numbers up to 10 = {1, 3, 5, 7, 9}

(*c*) A set of numbers divisible by 3 up to 10 = {3, 6, 9}.

1.4 SUBSETS

ACTIVITY 6

In groups, consider the following sets.

A = {all pupils of your class}

and B = {all girls of your class}

Discuss the following in your group.

1. What is the relationship between set B and set A?
2. Is it true that members of set B are the members of set A?

Consider the sets:

X = {all pupils in your school}

Y = {all pupils in your class}

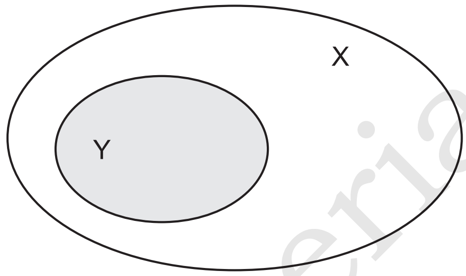
Here every member of set Y is also a member of set X. Therefore, set Y is a subset of set X.

Set Y is a subset of set X is expressed in symbols as $Y \subset X$ or $X \supset Y$. We read it as all elements of set Y are contained in set X.

Using Venn diagram we represent it as (see the adjacent figure):

The symbol '⊂' stands for '*is a subset of*' or '*is contained in*'. If Y is not a subset of X, we write $Y \not\subset X$.

Note: Every set is a subset of itself. Since the empty set ϕ has no element, ϕ is also a subset of every set. Therefore, $A \subset A$ and $\phi \subset A$.



Y ⊂ X

Example 10. *If X = {a, b, c}, then find its subsets.*

Solution. The subsets are:

{ }, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*b*, *c*}, {*c*, *a*}, {*a*, *b*, *c*}.

Set Notation

The symbols used in set notation are:

Symbols	Meaning	Example
∈	belongs to (is an element of)	$b \in \{a, b, c, d, e\}$
∉	does not belong to (is not an element of)	$m \notin \{a, b, c, d, e\}$
⊂	is a subset of (is contained in)	$\{a, b, c\} \subset \{a, b, c, d, e\}$
⊄	is not a subset of (is not contained in)	$\{x, y, z\} \not\subset \{a, b, c, d, e\}$
U	universal set	If A = {*a*, *b*, *c*}, B = {*c*, *d*, *e*}, then U = {*a*, *b*, *c*, *d*, *e*}
{ } or ϕ	null or empty set	A set of months with 32 days is a null set.
=	equal to	If A = {*a*, *b*, *c*}, B = {*c*, *a*, *b*}, then A = B
n(A)	number of elements in the set A	If A = {1, 2, 3, 4, 5}, then *n*(A) = 5

EXERCISE 1.2

1. State as equivalent and equal sets.

(*a*) A = {8, 9, 15} and B = {8, 9, 15}

(*b*) C = {*a*, *d*, *e*, *f*} and D = {1, 2, 3, 4, 5}

(*c*) P = {7, 9, 11, 15} and Q = {15, 11, 9, 7}

(*d*) P = {even numbers less than 10} and
Q = {odd numbers greater than 10}

2. Fill in the following blanks with 'empty set' or 'non-empty set' in your notebook.

(*a*) A cat with six legs ____________

(*b*) A year with fifteen months ____________

(*c*) A car that uses water for fuel ____________

(*d*) Pupils who are owning an airplane ____________

(*e*) A fish that flies ____________

(*f*) A school whose pupils do not study ____________

3. Fill in the following blanks with 'Finite set' or 'Infinite set' in your notebook.

(*a*) P = {5, 10, 15, 20, 25 30} ____________

(*b*) W = {0, 1, 2, 3, ...} *i.e.* set of all counting numbers. ____________

(*c*) N = {1, 2, 3, ...} *i.e.* set of all natural. ____________

(*d*) Q = {natural numbers less than 25} ____________

(*e*) R = {whole numbers between 5 and 45} ____________

4. Given universal set U = {1, 2, 3, 4, ..., 30}.
Write the following sets and represent them using Venn diagrams.

(*a*) A set of numbers less than 10

(*b*) A set of numbers between 10 and 20

(*c*) A set of numbers greater than 6

5. If X = {*a*, *b*}, then find the number of subsets of X.

6. Write the subsets of the set {1, 2, 3}.

7. Consider the sets ϕ, A = {1, 4}, B = {1, 3, 9}, C = {1, 3, 4, 5, 7, 9}.
Insert the symbol $\subset$ or $\not\subset$ between each of the following pairs of sets:

(*a*) ϕ ... B (*b*) A ... B (*c*) A ... C (*d*) B ... C

8. Suppose A = {*a*, *e*, *i*, *o*, *u*} and B = {*a*, *b*, *c*, *d*}. Is A a subset of B? Why? Is B a subset of A? Why?

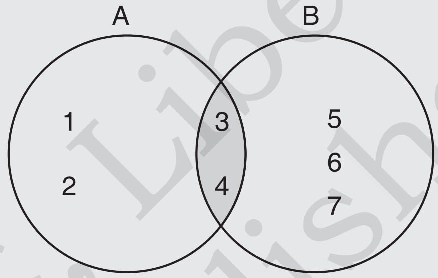
1.5 VENN DIAGRAMS TO ILLUSTRATE INTERSECTION (∩) OF SETS

ACTIVITY 7

In a group of 4 pupils, observe the following Venn diagram.

Discuss the following:

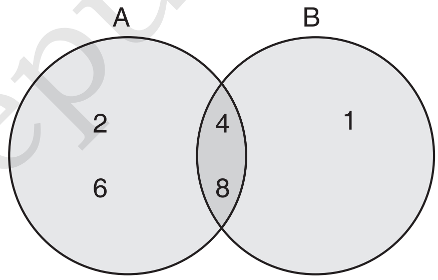
1. What are the elements of set A?
2. What are the elements of set B?
3. What are the elements of set A and set B if combined together?
4. What are the common elements of set A and set B?



The intersection of two sets A and B is the set formed by putting the common elements of these two sets together. The symbol 'Ç' means 'intersection'.

For example:

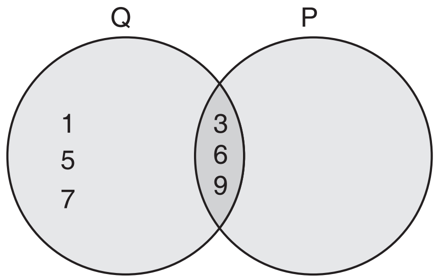
If A = {2, 4, 6, 8} and B = {1, 4, 8}, then A ∩ B = {4, 8}, since 4 and 8 belong to both sets A and B. The shaded region in the figure is the intersection of sets A and B.



Example 11. *If P = {3, 6, 9} and Q = {1, 3, 5, 6, 7, 9}, then find P ∩ Q.*

Solution. We have P ∩ Q = {3, 6, 9}

Using Venn diagram it is represented as (see the adjacent figure).



1.6 VENN DIAGRAMS TO ILLUSTRATE UNION (∪) OF SETS

The union of two sets A and B is a set formed by putting the elements of two sets together. The symbol ∪ means 'union'.

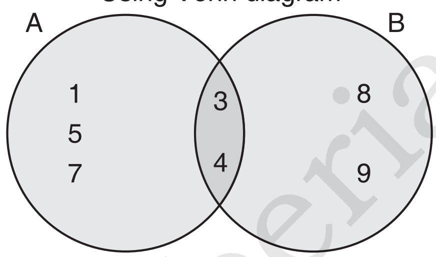
For example:

If A = {1, 3, 4, 5, 7} and B = {3, 4, 8, 9}, then

$$A \cup B = \{1, 3, 4, 5, 7, 8, 9\}$$

Using Venn diagram it is represented as (see the adjacent figure):

Using Venn diagram



Note: If a number appears in both sets, it is written once.

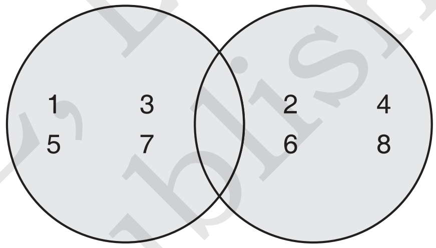
Example 12. *If set A = {1, 3, 5, 7} and B = {2, 4, 6, 8}, find A ∪ B.*

Solution. The union of sets A and B, that is,

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

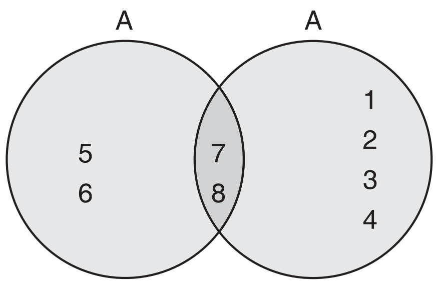
Using Venn diagram it is represented as (see the adjacent figure):

Venn diagram



EXERCISE 1.3

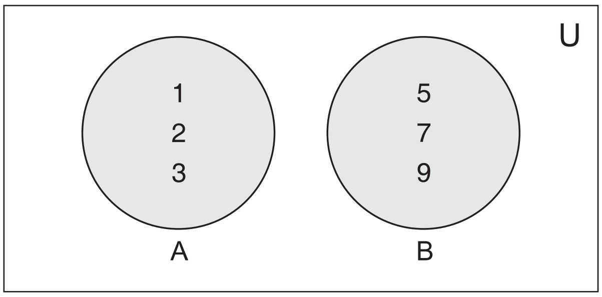
1. If Q = {4, 8, 12, 16} and R = {12, 13, 14}, find Q ∩ R.
2. Find B ∩ C if B = {even numbers less than 10} and C = {multiples of 3 less than 10}
3. If A = {1, 3, 4, 5, 7} and B = {3, 4, 8, 9}, find A ∩ B.
4. If Q = { 7, 8, 9, 10} and R = {5, 6, 7, 8}, find Q ∪ R.
5. If X = {prime numbers less than 13} and Y = {odd natural numbers less than 13}, then
   (*a*) list the members of sets X and Y.
   (*b*) list the members of X ∪ Y.
6. If P = {prime numbers less than 20} and Q = {odd natural numbers less than 10}, find P ∪ Q.
7. Study the given Venn diagram and answer the following questions.
   (*a*) Find the members of sets A and B.
   (*b*) Find A ∪ B.



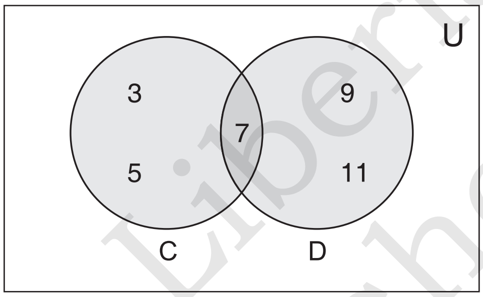
Venn diagram

1.7(A) VENN DIAGRAMS TO SHOW DISJOINT SETS

Two sets A and B are said to be disjoint, if they have no element in common.



Disjoint Sets



Not Disjoint Sets

ACTIVITY 8

Consider the activity 1 again.

The first group is a group of girls.

The second group is a group of boys.

Is there any pupil who is common in both the groups?

Example 13. *If A = {4, 6, 10} and B = {7, 11, 15}, then find A ∩ B. Are the sets A and B disjoint sets? If not, why?*

Solution. Here, $A \cap B = \phi$

As there is no common element in A and B, therefore, these are disjoint sets.

1.7(B) VENN DIAGRAMS TO SHOW COMPLEMENT OF A SET

ACTIVITY 9

Work in pairs. Consider A = {2, 4, 6, 8, 10}; B = {1, 3, 5, 7, 9} and U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

1. Find (*i*) U – A (*ii*) U – B
2. What do you call the new sets you get?

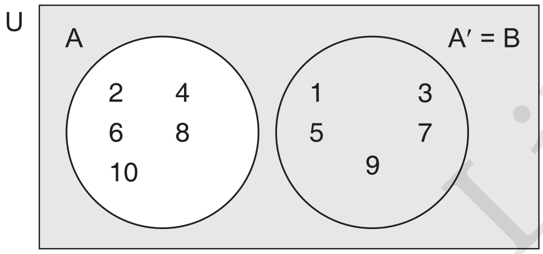
In this activity, set A has all the elements of the universal set U which are not in set B. Similarly, set B has all the elements of the universal set U which are not in set A.

These two sets complement each other with respect to set U.

Symbolically, we write A′ or C_A^U to denote the *complement of A* or A′ = U – A.

Here, A′ = U – A = {1, 3, 5, 7, 9} = B

Let us use Venn diagram to represent A′.



Shaded area in the universal set represents complement of set A.

Example 14. *Find A′ for A = {3, 6, 7, 8}, where the universal set U is given by*

(*a*) *U = {1, 2, 3, …, 8}* (*b*) *U = {2, 3, 4, …, 10}*

Solution. We have A = {3, 6, 7, 8}

(*a*) U = {1, 2, 3, …, 8}

∴ A′ = U – A = Elements of set U which are not in A

= **{1, 2, 4, 5}**

(*b*) U = {2, 3, 4, …, 10}

∴ A′ = U – A = {2, 3, 4, …, 10} – {3, 6, 7, 8}

= **{2, 4, 5, 9, 10}**

Example 15. *If the universal set U = {7, 14, 21, 28, 35}, find C_A^U, where*

(*a*) *A = {21, 35}* (*b*) *A = {first two multiples of 7}*

Solution. We have U = {7, 14, 21, 28, 35}

(*a*) A = {21, 35}

∴ C_A^U = U – A = {7, 14, 21, 28, 35} – {21, 35}

= **{7, 14, 28}**

(*b*) A = {first two multiples of 7} = {7, 14}

∴ C_A^U = U – A = {7, 14, 21, 28, 35} – {7, 14}

= **{21, 28, 35}**

Example 16. *If A = {2, 4, 6}; B = {2, 3, 5} and the universal set U = {1, 2, 3, 4, 5, 6, 7, 8}, find (A ∪ B)′ and (A ∩ B)′.*

Solution. $A \cup B = \{2, 4, 6\} \cup \{2, 3, 5\} = \{2, 3, 4, 5, 6\}$

$\therefore \quad (A \cup B)' = U - (A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 3, 4, 5, 6\}$

$= \mathbf{\{1, 7, 8\}}$

Now, $A \cap B = \{2, 4, 6\} \cap \{2, 3, 5\} = \{2\}$

$\therefore \quad (A \cap B)' = U - (A \cap B) = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2\}$

$= \mathbf{\{1, 3, 4, 5, 6, 7, 8\}}$

EXERCISE 1.4

1. Show that set A = {2, 5, 6} and set B = {4, 7, 8} are disjoint sets.
2. Are the set P = {3, 8, 9} and set Q = {9, 10, 11}, disjoint sets? If not, justify your answer.
3. State whether $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets or not.
4. If the universal set is U = {1, 2, 3, ..., 10}, find A′ where
   (*a*) A = {1, 2, 3, 4, 5} (*b*) A = {1, 7, 10}
   (*c*) A = {1} (*d*) A = {9}
   Represent them using Venn diagrams.
5. Given the set of natural numbers as the universal set, write down the complements of the following sets.
   (*a*) {even natural numbers}
   (*b*) the set of odd natural numbers
   (*c*) {factor of 2}
   (*d*) the set of natural numbers each more than 19
6. Given universal set $U = \{1, 2, 3, 4, 5, a, b, c, d, e\}$, find the complements of the following sets. Represent them using Venn diagrams.
   (*a*) A = {1, 2, 3, 4, 5} (*b*) $B = \{a, b, c, d, e\}$
7. Given universal set U = {0, 2, 4, 6, 8, 10, ..., 100}, which of the following pairs are the complements of each other?
   (*a*) {2, 4, 6, 8, 10} and {12, 14, 16, 18, ..., 100}
   (*b*) {0} and {2, 4, 6, 8, 10, ..., 100}
8. If X = {0, 2, 4, 6}, Y = {2, 4, 8, 16} and universal set U = {0, 2, 4, 6, 8, 10, 12, 14, 16}, then find
   (*a*) X' (*b*) Y' (*c*) $X \cap Y$

1.8 PROPERTIES OF SETS

1. Commutative Property (*Commutativity*)

Union and intersection of sets satisfy the commutative property.

For any two sets A and B,

$A \cup B = B \cup A$ (Commutative Property of Union)

$A \cap B = B \cap A$ (Commutative Property of Intersection)

For example:

(*a*) Consider A = {1, 2, 3, 4} and B = {2, 4, 6}, then

$A \cup B = \{1, 2, 3, 4\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 6\}$

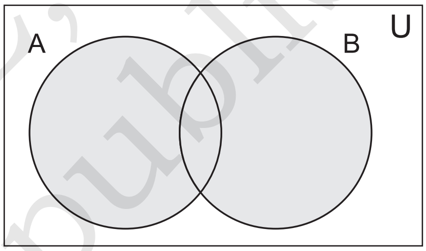
$B \cup A = \{2, 4, 6\} \cup \{1, 2, 3, 4\} = \{2, 4, 6, 1, 3\}$

We observe that $A \cup B$ and $B \cup A$ have exactly the same elements.

Moreover, the elements in a set can be listed in any order.

Therefore, $A \cup B = B \cup A$

The result is true for any two sets. (See the Venn diagram).



A ∪ B (Shaded) = B ∪ A (Shaded)

The Venn diagrams for A ∪ B and B ∪ A are identical (See the Venn diagram).

Thus, union of sets is commutative.

(*b*) Consider A = {*a*, *b*, *c*, *d*, *e*} and B = {*a*, *e*, *i*, *o*, *u*}. A has elements *a* and *e* common with B and B has elements *a* and *e* common with A.

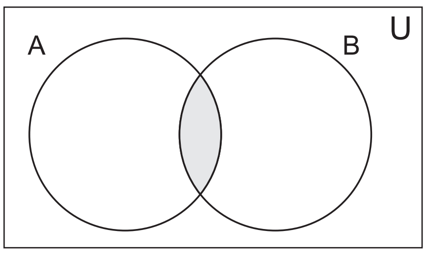
Thus, $A \cap B = \{a, b, c, d, e\} \cap \{a, e, i, o, u\} = \{a, e\}$

$B \cap A = \{a, e, i, o, u\} \cap \{a, b, c, d, e\} = \{a, e\}$

We observe that $A \cap B$ and $B \cap A$ have exactly the same elements.

Therefore, $A \cap B = B \cap A$

The result is true for any two sets (See the Venn diagram).



A ∩ B (Shaded) = B ∩ A (Shaded)

The Venn diagrams for A ∩ B and B ∩ A are identical (See the Venn diagram).

Thus, intersection of sets is commutative.

2. Associative Property (*Associativity*)

Union and intersection of sets satisfy the associative property.

For any three sets A, B and C,

$(A \cup B) \cup C = A \cup (B \cup C)$ (Associative Property of Union)

$(A \cap B) \cap C = A \cap (B \cap C)$ (Associative Property of Intersection)

For example:

(*a*) Consider $A = \{2, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{1, 4, 7\}$

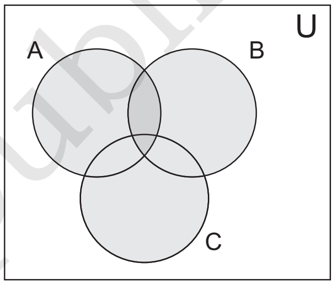
Then $A \cup B = \{2, 3, 5\} \cup \{2, 4, 6\} = \{2, 3, 5, 4, 6\}$

and $(A \cup B) \cup C = \{2, 3, 5, 4, 6\} \cup \{1, 4, 7\} = \{2, 3, 5, 4, 6, 1, 7\}$

Also, $B \cup C = \{2, 4, 6\} \cup \{1, 4, 7\} = \{2, 4, 6, 1, 7\}$

and $A \cup (B \cup C) = \{2, 3, 5\} \cup \{2, 4, 6, 1, 7\} = \{2, 3, 5, 4, 6, 1, 7\}$

We observe that $(A \cup B) \cup C$ and $A \cup (B \cup C)$ have exactly the same elements. Moreover, the elements in a set can be listed in any order.



$(A \cup B) \cup C$ (Shaded) $= A \cup (B \cup C)$ (Shaded)

Therefore, $(A \cup B) \cup C = A \cup (B \cup C)$

The result is true for any three sets.

The Venn diagrams for $(A \cup B) \cup C$ and $A \cup (B \cup C)$ are identical (See the diagram).

Thus, union of sets is associative.

(*b*) Consider $A = \{a, b, c, d\}$, $B = \{b, d, f, g\}$ and $C = \{c, d, e, f\}$

Then, $A \cap B = \{a, b, c, d\} \cap \{b, d, f, g\} = \{b, d\}$

and $(A \cap B) \cap C = \{b, d\} \cap \{c, d, e, f\} = \{d\}$

Also, $B \cap C = \{b, d, f, g\} \cap \{c, d, e, f\} = \{d, f\}$

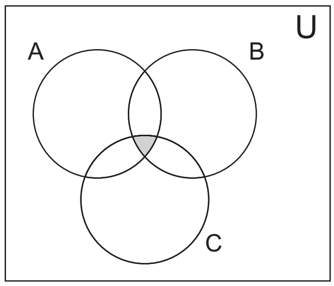
and $A \cap (B \cap C) = \{a, b, c, d\} \cap \{d, f\} = \{d\}$

Clearly, $(A \cap B) \cap C = A \cap (B \cap C)$

The results is true for any three sets.

The Venn diagrams for $(A \cap B) \cap C$ and $A \cap (B \cap C)$ are identical (See the diagram).

Thus, intersection of sets is associative.



$(A \cap B) \cap C$ (Shaded) $= A \cap (B \cap C)$ (Shaded)

3. Distributive Property (*Distributivity*)

Union and intersection of sets satisfy the distributive property.

For any three sets A, B and C,

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(Union distribute over intersection)

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(Intersection of distribute over union)

For example:

(*a*) Consider A = {1, 2, 3, 4}; B = {2, 4, 6, 8} and C = {3, 4, 5, 6}.

Then, $B \cap C = \{2, 4, 6, 8\} \cap \{3, 4, 5, 6\} = \{4, 6\}$

and $A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{4, 6\} = \{1, 2, 3, 4, 6\}$ ...(1)

Also, $A \cup B = \{1, 2, 3, 4\} \cup \{2, 4, 6, 8\} = \{1, 2, 3, 4, 6, 8\}$

and $A \cup C = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$

so that

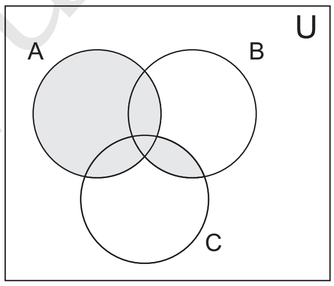
$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\} = \{1, 2, 3, 4, 6\}$ ...(2)

From (1) and (2), we conclude that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Thus, the result is true for any three sets.

The Venn diagrams for $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$ are identical (see the diagram).



A ∪ (B ∩ C (Shaded)
= (A ∪ B) ∩ (A ∪ C) (Shaded)

Thus, union distributes over intersection.

(*b*) Consider A = {*b*, *d*, *e*}, B = {*a*, *b*, *c*} and C = {*c*, *d*, *f*}.

Then, $B \cup C = \{a, b, c\} \cup \{c, d, f\} = \{a, b, c, d, f\}$

and $A \cap (B \cup C) = \{b, d, e\} \cap \{a, b, c, d, f\} = \{b, d\}$ ...(1)

Also, $A \cap B = \{b, d, e\} \cap \{a, b, c\} = \{b\}$

and $A \cap C = \{b, d, e\} \cap \{c, d, f\} = \{d\}$

so that $(A \cap B) \cup (A \cap C) = \{b\} \cup \{d\} = \{b, d\}$ ...(2)

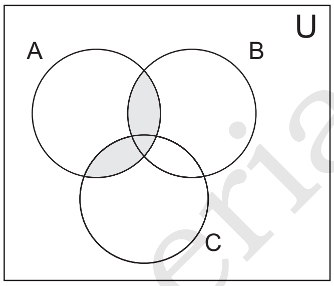
From (1) and (2), we conclude that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Thus, the result is true for any three sets (See the diagram).

The Venn diagrams for $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$ are identical (See the diagram).

Thus, intersection distributes over union.



$A \cap (B \cup C)$ (Shaded)
$= (A \cap B) \cup (A \cap C)$ (Shaded)

4. Some More Properties of Operations on Sets

(*i*) $A \cup \phi = A$ (*ii*) $A \cap \phi = \phi$ (*iii*) $A \cup A = A$

(*iv*) $A \cap A = A$ (*v*) $A \cup U = U$ (*vi*) $A \cap U = A$

(*vii*) $A \subset A \cup B$ (*viii*) $B \subset A \cup B$ (*ix*) $A \cap B \subset A \cup B$

(*x*) $A \cap B \subset A$ (*xi*) $A \cap B \subset B$

(*xii*) If $A \subset B$, then $A \cup B = B$ and $A \cap B = A$

Example 17. *If* $A = \{1, 2, 3, 5\}$, $B = \{2, 4, 5\}$ *and* $C = \{3, 4, 6\}$, *find*

(*i*) $A \cup B$ (*ii*) $B \cap C$

(*iii*) $(A \cap B) \cup C$ (*iv*) $A \cap (B \cup C)$

Solution. (*i*) $A \cup B = \{1, 2, 3, 5\} \cup \{2, 4, 5\}$

= Set of all elements of A and B, droping repetitions

$= \{1, 2, 3, 4, 5\}$

(*ii*) $B \cap C = \{2, 4, 5\} \cap \{3, 4, 6\}$

= Set of common elements of A and B = $\{4\}$

(*iii*) $A \cap B = \{1, 2, 3, 5\} \cap \{2, 4, 5\} = \{2, 5\}$

$(A \cap B) \cup C = (2, 5\} \cup \{3, 4, 6\}$

$= \{2, 3, 4, 5, 6\}$

(*iv*) $B \cup C = \{2, 4, 5\} \cup \{3, 4, 6\} = \{2, 3, 4, 5, 6\}$

$A \cap (B \cup C) = \{1, 2, 3, 5\} \cap \{2, 3, 4, 5, 6\} = \{2, 3, 5\}$

Example 18. *If* $A = \{a, b, c\}$, $B = \{b, c, d, e\}$ *and* $C = \{a, d, f\}$. *Verify that* $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Solution. Here $B \cup C = \{b, c, d, e\} \cup \{a, d, f\} = \{a, b, c, d, e, f\}$

$A \cap (B \cup C) = \{a, b, c\} \cap \{a, b, c, d, e, f\} = \{a, b, c\}$ ...(1)

Also, $A \cap B = \{a, b, c\} \cap \{b, c, d, e\} = \{b, c\}$

$A \cap C = \{a, b, c\} \cap \{a, d, f\} = \{a\}$

$(A \cap B) \cup (A \cap C) = \{b, c\} \cup \{a\} = \{a, b, c\}$ ...(2)

From (1) and (2), we have

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

EXERCISE 1.5

1. Let A = {1, 2, 3, 4}, B = {2, 3, 5, 7}, C = {3, 6, 7, 8} and D = {3, 5, 7, 9}; find

(*a*) $A \cup B$ (*b*) $B \cup C$ (*c*) $C \cup D$ (*d*) $A \cap B$

(*e*) $B \cap C$ (*f*) $C \cap D$ (*g*) $A \cup (B \cap D)$ (*h*) $B \cap (A \cup C)$

2. If A = {1, 2, 3, 4, 5, 6}, B = {2, 4, 6, 8} and C = {3, 5, 6, 8}, verify that

(*a*) $A \cup B = B \cup A$ (*b*) $B \cap C = C \cap B$

(*c*) $(A \cup B) \cup C = A \cup (B \cup C)$ (*d*) $(A \cap B) \cap C = A \cap (B \cap C)$

3. Let A = {2, 3, 4} and B = {2, 3, 4, 5, 6}. Use Venn diagram to show:

(*a*) $A \subset B$ (*b*) $A \cap B = A$ (*c*) $A \cup B = B$

4. Let A = {*x* : *x* is a letter of the word PERMANENT}

and B = {*x* : *x* is a letter of the word TEMPORARY}

(*a*) Find $A \cup B$ (*b*) Find $A \cap B$

(*c*) Verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

5. Let A = {*x* : *x* is an even natural number}

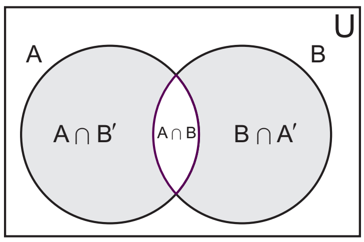
B = {*x* : *x* is an odd natural number}

and C = {*x* : *x* is a prime number}

Find (*a*) $A \cap B$ (*b*) $A \cup B$ (*c*) $A \cap C$.

1.9 VENN DIAGRAMS TO SOLVE TWO-SET *AND* THREE-SET PROBLEMS

If A and B are two finite sets, then it is clear from the given Venn diagram that



(*i*) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(*ii*) $n(A \cap B') = n(A) - n(A \cap B)$

(*iii*) $n(B \cap A') = n(B) - n(A \cap B)$

(*iv*) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

If A, B and C are three finite sets, then

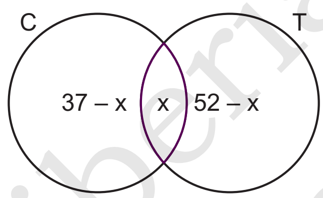
(*i*) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(*ii*) $n(A \cup B \cup C) = n(A) + n(B) + n(C)$ [if the sets are mutually disjoint]

Example 19. *In a group of 70 persons, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many person like (i) both coffee and tea? (ii) coffee and not tea? (iii) tea and not coffee?*

Solution. Let C = Set of persons who like coffee

T = Set of persons who like tea



Suppose x persons like both coffee and tea.

Then,

$n(C \cap T) = x$. Write x in the region common to C and T.

$n(C) = 37 \Rightarrow (37 - x)$ like coffee and not tea. Write $(37 - x)$ in the remaining part of C.

$n(T) = 52 \Rightarrow (52 - x)$ like tea and not coffee. Write $(52 - x)$ in the remaining part of T as shown in the diagram.

Since each person likes at least one of the two drinks, $n(C \cup T) = 70$

$$\Rightarrow (37 - x) + x + (52 - x) = 70$$

$$\Rightarrow 37 + 52 - x + x - x = 70$$

$$\Rightarrow 89 - x = 70$$

$$\Rightarrow -x = 70 - 89$$

$$\Rightarrow -x = -19 \Rightarrow x = 19$$

Therefore, 19 persons like both coffee and tea.

Number of persons who like coffee and not tea

$$= n(C \cap T') = 37 - x = 37 - 19 = 18$$

Number of persons who like tea and not coffee

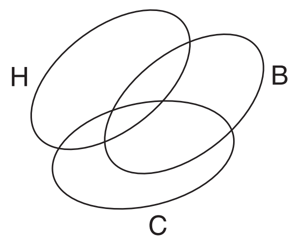
$$= n(T \cap C') = 52 - x = 52 - 19 = 33$$

Example 20. *In a class of 60 pupils, 23 play Hockey, 15 play Basketball and 20 play Cricket. 7 play Hockey and Basketball, 5 play Cricket and Basketball, 4 play Hockey and Cricket and 15 pupils do not play any of these games. Find:*

(a) how many play Hockey, Basketball and Cricket?

(b) how many play Hockey but not Cricket?

(c) how many play Hockey and Cricket but not Basketball?



Solution. Let H = set of pupil playing Hockey

B = set of pupil playing Basketball

and C = set of pupil playing Cricket

$\therefore \quad n(H) = 23,\ n(B) = 15,\ n(C) = 20,$

$n(H \cap B) = 7,\ n(C \cap B) = 5,$

$n(H \cap C) = 4.$

$n(H' \cap B' \cap C') = 15.$

$\therefore \quad n(H \cup B \cup C) = 60 - n(H' \cap B' \cap C') = 60 - 15 = 45.$

(*a*) We have

$$n(H \cup B \cup C) = n(H) + n(B) + n(C) - n(H \cap B) - n(B \cap C) - n(H \cap C) + n(H \cap B \cap C)$$

$\Rightarrow \quad 45 = 23 + 15 + 20 - 7 - 5 - 4 + n(H \cap B \cap C)$

$\Rightarrow \quad 45 = 42 + n(H \cap B \cap C)$

$\Rightarrow \quad n(H \cap B \cap C) = 45 - 42 = 3$

$\therefore$ No. of pupils playing Hockey, Basketball and Cricket

$= n(H \cap B \cap C) = 3$

(*b*) No. of pupils playing Hockey but not Cricket

$= n(H - C) = n(H) - n(H \cap C) = 23 - 4 = \mathbf{19}$

(*c*) No. of pupils playing Hockey and Cricket but not Basketball

$n((H \cap C) - B) = n(H \cap C) - n(H \cap C \cap B)$

$= 4 - 3 = \mathbf{1}.$

EXERCISE 1.6

1. Out of 500 car owners investigated, 400 owned cars A and 200 owned cars B; 50 owned both A and B cars. Is this data correct?

2. In a survey of 600 pupils in a school, 150 pupils were found to be drinking Tea and 225 drinking Coffee, 100 were drinking both Tea and Coffee. Find how many pupils were drinking neither Tea nor Coffee.

3. A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number of consumers that must have liked both products?

4. There are 20 pupils in a Chemistry class and 30 pupils in a Physics class. Find the number of pupils studying either Chemistry or Physics in the following cases:

(*a*) the two classes meet at the same hour

(*b*) the two classes meet at different hours and ten pupils are enrolled in both the courses.

5. Of the number of three athletic teams in a school, 21 are in the basketball team, 26 in hockey team and 29 in the football team. 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the games. How many members are there in all?

REVIEW EXERCISE

1. Which of the following are sets?

(*a*) {odd prime numbers}

(*b*) {natural numbers between 40 and 50 not divisible by 2}

(*c*) A collection of all boys in the class

(*d*) {*a*, *b*, *c*, *d*, *e*, *f*}

(*e*) The collection of letters of the word "mathematics"

2. Write down all the elements of the following sets.

(*a*) {the numbers between 10 and 30 not divisible by 3}

(*b*) {the prime numbers between 10 and 50}

(*c*) A set of prime numbers divisible by 3

3. Determine the cardinality of each of the following sets.

(*a*) {all natural numbers}

(*b*) {all prime numbers less than 2}

(*c*) A set of 2-digit numbers divisible by both 3 and 5

4. Copy and fill in the blanks using the symbol $\in$ or $\notin$.

(*a*) 12 ___ {prime numbers} (*b*) 15 ___ {odd prime numbers}

(*c*) 1 ___ {prime numbers} (*d*) 16 ___ {even numbers}

5. State as equivalent and equal sets.

(*a*) M = {*m*, *a*, *t*, *h*, *e*, *i*, *c*, *s*} and N = {*e*, *n*, *g*, *l*, *i*, *s*, *h*}

(*b*) A = {*a*, *e*, *i*, *o*, *u*} and B = {*o*, *u*, *i*, *a*, *e*}

(*c*) A = {10, 20, 30, 40, 50, 60} and B = {70, 80, 90, 100}

6. Fill in the following blanks with 'empty set' or 'non-empty set' in your notebook.

(*a*) F = {A week having nine days} ____________

(*b*) Q = {Sheep which produce milk} ____________

(*c*) R = {A boy who is as old as his father} ____________

(*d*) X = {A mother of fifty years of age} ____________

(*e*) N = {Fish with wings} ____________

(*f*) Z = {Car which uses diesel as fuel} ____________

7. Fill in the following blanks with 'Finite set' or 'Infinite set' in your notebook.

(*a*) Set of all points in a plane. ____________

(*b*) Set of all positive integers which is multiple of 3. ____________

(*c*) The set of all persons in Liberia. ____________

(*d*) Set of all points in a line segment. ____________

(*e*) The set of all birds in California. ____________

8. Which of the following is the universal set of the other two?

(*a*) A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8}, C = {1, 2, 3, 4, 5, 6, 7, 8, 9}

(*b*) A = {P, Q, R, S, T}, B = {P}, C = {Q, R, S, T}

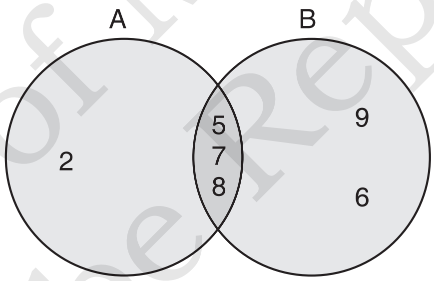
(*c*) A = {pig, goat}, B = {goat, cat, pig, dog}, C = {pig, cat, dog}

9. Is {all natural numbers} the universal set of {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...} and {0}? Why?

10. Is the set C the universal set of sets A and B if
A = {2, 4, 6, 8}, B = {factors of 12}, C = {2, 3, 4, 6, 8, 12}? Why?

11. If P = {7, 9, 13} and Q = {1, 7, 13}, find $P \cap Q$.

12. Study the Venn diagram given below and find $A \cap B$.



13. List the members of each of the sets B = {natural numbers from 20 to 30} and D = {15, 16, 20, 21, 25, 26, 28} and find $B \cup D$.

14. If the universal set is U = {1, 2, 3, …, 10}, find A′ where

(*a*) A = {1, 2, 3, …, 9} (*b*) A = {2, 4, 6, 8, 10}

(*c*) A = {odd numbers up to 9} (*d*) A = {factors of 10}

15. Given universal set U = {1, 2, 3, 4, 5, *a*, *b*, *c*, *d*, *e*}, find the complements of the following sets. Represent them using Venn diagrams.

(*a*) P = {1, 2, 3, *a*, *b*, *c*} (*b*) Q = {4, 5, *d*, *e*}

16. Given universal set U = {0, 2, 4, 6, 8, 10, …, 100}, which of the following pairs are the complements of each other?

(*a*) {multiples of 2 up to 100} and {0}

(*b*) {100, 98, 96, 94, …, 50} and {0, 2, 4, 6, 8, 10, …, 48}

17. If X = {0, 2, 4, 6}, Y = {2, 4, 8, 16} and universal set U = {0, 2, 4, 6, 8, 10, 12, 14, 16}, then find

(*a*) $X \cup Y$ (*b*) $(X \cap Y)'$ (*c*) $(X \cup Y)'$

18. Let A = {1, 2, 3, 4}, B = {2, 3, 5, 7}, C = {3, 6, 7, 8} and D = {3, 5, 7, 9}; find

(*a*) $(A \cup B) \cap (B \cap D)$ (*b*) $(B \cap C) \cup A$.

19. If A = {1, 2, 3, 4, 5, 6}, B = {2, 4, 6, 8} and C = {3, 5, 6, 8}, verify that

(*a*) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (*b*) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

20. There are 200 individuals with a skin disorder. 120 had been exposed to chemical C_1, 50 to chemical C_2, and 30 to both chemicals C_1 and C_2. Find the number of individuals exposed to:

(*a*) chemical C_1 but not chemical C_2

(*b*) chemical C_2 but not chemical C_1

(*c*) chemical C_1 or chemical C_2.

21. Each pupils in a class of 40, studies at least one of the subjects English, Mathematics and Economics. 16 study English, 22 Economics and 26 Mathematics, 5 study English and Economics, 14 Mathematics and Economics and 2 English, Economics and Mathematics. Find the number of pupils who study:

(*a*) English and Mathematics

(*b*) English, Mathematics but not Economics.

MULTIPLE CHOICE QUESTIONS (MCQs)

1. Set A is called ...... of sets B when all the members of set A are also members of set B.

(*a*) the universal set (*b*) the union set

(*c*) the null set (*d*) a subset

2. Two sets which have no common element(s) are known as

(*a*) equal sets (*b*) unequal (*c*) empty sets (*d*) disjoints sets

3. A = {1, 2, 3, 8, 9} and B = {8, 1, x, 3, 2}. If A = B. What is the value of x.

(*a*) 2 (*b*) 3 (*c*) 9 (*d*) 1

4. A = {0, 2, 4, 6} and B = {1, 2, 4, 5}. Find $A \cap B$.

(*a*) {0, 6} (*b*) {2, 4} (*c*) {0, 2, 4} (*d*) {0, 2, 6}

5. Which of the following is true?

(*a*) {2, 3, 5, 7, 27} is a subset of prime numbers.

(*b*) {1, 0, 2, 3, 5} is a subset of odd numbers.

(*c*) {– 2, – 1, 1, 3, 9} is a subset of integers.

(*d*) {0, 2, 6, 9, 12} is a subset of even numbers.

6. P = {Multiples of 3 between 10 and 20}, Q = {even numbers between 10 and 20}. Find P ∩ Q.

(*a*) {12, 18} (*b*) {12, 14, 16, 18}
(*c*) {12, 15, 18} (*d*) {12, 14, 15, 16}

7. R = {1, 3, 5, 7} and S = {2, 4, 6, 8}. Find R ∪ S.

(*a*) {1, 2, 3, 5, 6, 8} (*b*) { }
(*c*) {1, 2, 3, 4, 5, 6, 7, 8} (*d*) {1, 2, 3, 4, 5, 7, 8}

8. If A = {1, 3, 5, 7, 11, 13, 15} and B = {1, 2, 3, 5, 6, 7, 10, 11, 12}. Find A ∩ B.

(*a*) {1, 3, 5, 7, 9, 11} (*b*) {2, 4, 8, 9, 13, 14}
(*c*) {1, 2, 3, 5, 6, 7, 10, 11, 12} (*d*) {1, 3, 5, 7, 11}

9. P = {*g*, *o*, *q*, *s*} and Q = {*h*, *p*, *r*, *t*}. Find P ∪ Q.

(*a*) {*q*, *r*, *s*, *t*} (*b*) {*g*, *h*, *o*, *q*, *r*}
(*c*) {*g*, *h*, *o*, *p*, *q*, *r*, *t*} (*d*) {*g*, *h*, *o*, *p*, *q*, *r*, *s*, *t*}

10. If P = {multiples of 4 less than 16}, find P.

(*a*) {1, 4, 8, 12} (*b*) {4, 8, 2} (*c*) {4, 8, 10} (*d*) {4, 8, 12, 16}

RECAP AT A GLANCE

- A well-defined collection of objects is called a set.
- In a Venn diagram, the sets are represented by shapes; usually circles or ovals or rectangles.
- Sets having exactly the same members or elements are called *equal sets*.
- A set that has no member or element is called an *empty* or *null set*.
- A set with limited number of members or one whose last member is known is called a finite set.
- A set with *unlimited* number of elements or one whose last element is not known is called an *infinite set*.
- Every member of set Y is also a member of set X. Therefore, set Y is a subset of set X.
- The union of two sets A and B is a set formed by putting the elements of two sets together.
- Two sets A and B are said to be disjoint, if they have no element in common.
- Write A′ or C_A^U to denote the *complement of A* or A′ = U – A.
- Union and intersection of sets satisfy the commutative property.
- Union and intersection of sets satisfy the associative property.
- Union and intersection of sets satisfy the distributive property.

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